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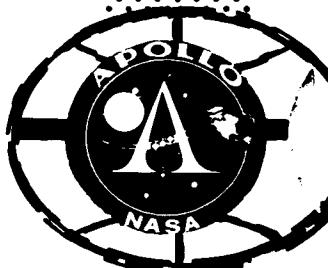
LM POSITION AND VELOCITY
UNCERTAINTIES AT CSI AS A
FUNCTION OF LUNAR LANDING
SITES AND MSFN TRACKING
GEOMETRY

By Paul H. Mitchell
Mathematical Physics Branch

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LM POSITION AND VELOCITY UNCERTAINTIES AT CSI AS A FUNCTION OF
LUNAR LANDING SITES AND MSFN TRACKING GEOMETRY

By Paul H. Mitchell

SUMMARY AND INTRODUCTION

A parametric study was made of the uncertainties in LM position and velocity at coelliptic sequence initiation (CSI) as a function of lunar landing site longitudes, lunar declinations, and Manned Space Flight Network (MSFN) tracking geometries. The results are plotted as root-sum-square (RSS) position and velocity uncertainties at CSI as a function of lunar landing site longitudes ranging from 45° E to 45° W for lunar declinations of 18° N, 0° , and 18° S and for three sets of MSFN tracking stations taking 2- and 3-way Doppler data.

This study allowed 10 minutes for orbit determination convergence, update, and prethrust programming between the last usable MSFN data and the CSI maneuver or LM occultation, whichever occurred earlier. For landing sites further west than 15° W, occultation was the controlling factor for CSI update. Using the 10-minute constraint, about 17 minutes of data are available from 45° E to 15° W, decreasing to 7 minutes at 45° W. In the event the primary navigation and guidance system (PNGS) fails, 10 minutes may be somewhat optimistic since the abort guidance system (AGS) has no digital uplink. However, since there are a variety of ways to expedite the update, the worst consequence of exceeding 10 minutes would probably be a delay of a few minutes in time of CSI execution.

A consideration of the many factors involved in an update, together with the results presented for the 45° W case, led to the conclusion that as the landing sites move west of the 0° meridian, MSFN support of the AGS for CSI increases in complexity and decreases in reliability. For the most westerly sites the situation is extremely difficult.

SYMBOLS

AGS	abort guidance system
ANT	Antigua Island
ASC	Ascension Island
BDA	Bermuda Island
CDH	constant delta height
CNB	Canberra, Australia
CRO	Carnarvon, Australia
CSI	coelliptic sequence initiation
GST	Goldstone, California
GUA	Guam Island
HAW	Hawaii Island
LGC	LM guidance computer
LM	lunar module
MAD	Madrid, Spain
MSFN	Manned Space Flight Network
PNGS	primary navigation and guidance system
RSS	root-sum-square
TPI	terminal phase initiation

ANALYSIS

The uncertainty in determining the LM ascent trajectory using MSFN tracking data depends primarily on the following factors:

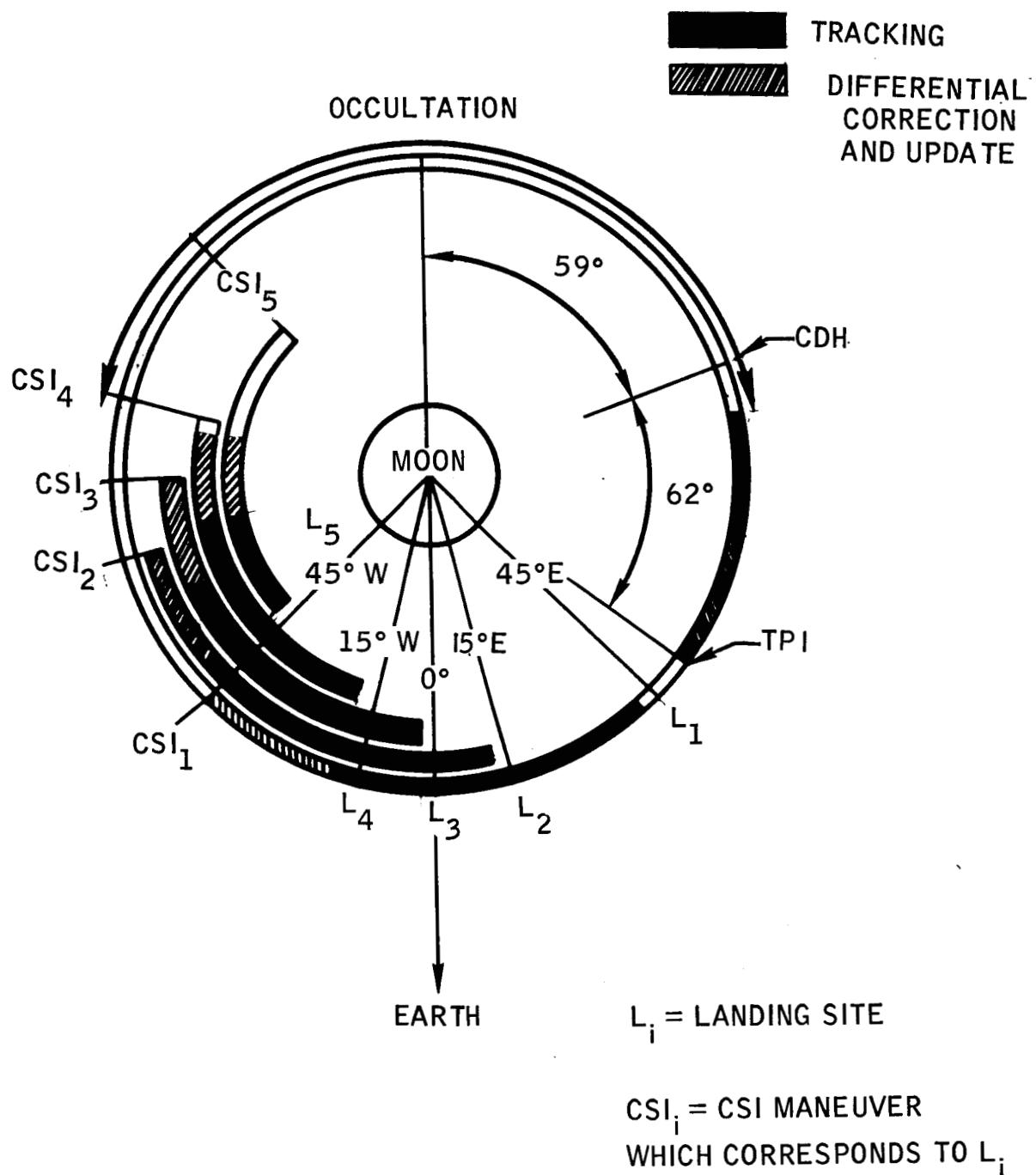
- a. The selenographic coordinates of CSI which vary with the coordinates of the landing site.
- b. The length of the permissible tracking arc between launch and CSI. The tracking arc is dependent on the coordinates of the landing site and the amount of time needed in real time to do a differential correction on the data and to implement an update of the LGC.
- c. The station-to-spacecraft geometry which varies with lunar declination and chosen tracking stations.
- d. The data characteristics such as biases, noise on the data, and the uncertainty in station location and the gravitational field.
- e. The a priori knowledge of the LM state at the termination of the main ascent burn.

The AS-50^{4A} preliminary reference trajectory (ref. 1) was used for this study. The line of nodes of the reference trajectory was rotated east or west, as applicable, so that the trajectory passed over the coordinates of the selected landing site. Also, the longitude of the moon was changed so that each of the following tracking configurations could be implemented in the analysis:

- | | |
|---|---|
| a. Canberra (primary)
Carnarvon (secondary)
Guam (secondary) | b. Madrid (primary)
Ascension (secondary)
Antigua (secondary) |
| c. Goldstone (primary)
Bermuda (secondary)
Hawaii (secondary) | |

The tracking arc for each of the landing sites was measured from launch burnout to the time of CSI minus 10 minutes or occultation minus 10 minutes, whichever occurred first. This 10 minutes allows time for orbit determination and LGC update.

The following schematic illustrates the geometry of the LM ascent-to-rendezvous sequence up to the initiation of the TPI maneuver for the various landing sites chosen for the study.



Assumptions

1. The uncertainties in the a priori knowledge of the LM at the time of ascent burnout were assumed to be 10 n. mi. in each component of position and 100 fps in each component of velocity.
2. The uncertainty in the gravitational constant of the moon was assumed to be $7.1 \times 10^9 \text{ ft}^3/\text{sec}^2$.
3. The data characteristics based on a sampling rate of one observation per 6 seconds for the MSFN tracking stations were assumed to be as follows:

Primary stations^a:

Bias, fps	0.03
Noise, fps	0.02

Secondary stations^b:

Bias, fps	0.2
Noise, fps	0.04

4. The average topocentric station location uncertainties were assumed to be as follows:

Altitude, ft	150
East-west component, ft	220
North-south component, ft	220

RESULTS

Tables I, II, and III contain the standard deviation in each component of position and velocity for the various cases considered in this study. The RSS of these standard deviations is plotted in figures 1 through 12. The data are plotted as a function of the longitude of lunar

^aNoise and bias were formulated from $\sigma(\dot{r}_1)$ where r_1 is the range from the station to the vehicle.

^bNoise and bias were formulated from $\sigma(\dot{r}_1 + \dot{r}_2)$ where r_1 is the range from the station to the vehicle and r_2 is the range from the vehicle to the station.

landing sites from 45° E to 45° W. Two presentations of the same data are made so that two comparisons can be made of the orbital accuracies at CSI. The first comparison can be made from figures 1 through 6 in which position and velocity uncertainties are plotted for three discrete lunar declinations. Each figure contains three curves, one for each of the three sets of MSFN tracking stations. This comparison shows the importance of the geometric configuration formed by the MSFN active tracking stations. In these cases the configurations are triangles since three stations are active simultaneously.

The second comparison can be made from figures 7 through 12 in which each of the MSFN station sets are plotted on separate figures. Each figure contains three curves identified for lunar declinations of 18° S, 0° , 18° N. This comparison shows the importance of the position of the moon relative to the MSFN stations.

CONCLUSIONS

A consideration of the many factors involved in an update, together with the results presented for the 45° W case, led to the conclusion that as the landing sites move west of the 0° meridian, MSFN support of the AGS for CSI increases in complexity and decreases in reliability. For the most westerly sites the situation is extremely difficult.

The results point out the importance of station-to-spacecraft geometry in orbit determination using MSFN tracking data. When tracking data from a primary station (2-way Doppler) and two secondary stations (3-way Doppler) are taken simultaneously, the three stations should be chosen so that their positions form a triangle whose vertices maximize east-west and north-south separations. The more important separation is in the north-south direction.

The general verification of these conclusions will depend on a future study in which the present study will be extended to include the rest of the rendezvous sequence.

TABLE I.- 3σ UNCERTAINTIES IN COMPONENTS OF POSITION AND VELOCITY
AT CSI BASED ON MAD, ASC, AND ANT TRACKING DATA

Landing site	3σ position uncertainty, ^a n. mi.			3σ velocity uncertainty, fps		
	$3\sigma_x$	$3\sigma_y$	$3\sigma_z$	$3\sigma_x^*$	$3\sigma_y^*$	$3\sigma_z^*$
(a) Lunar Declination = 18° N						
45° E	0.09	0.18	3.02	0.90	0.21	9.42
15° E	0.11	0.35	2.81	2.28	0.24	8.88
0°	0.19	0.38	2.26	2.94	0.48	6.93
15° W	0.26	0.32	1.40	3.09	0.69	3.78
45° W	3.30	2.81	0.54	31.92	9.99	10.14
(b) Lunar Declination = 0°						
45° E	0.09	0.18	2.89	0.90	0.21	9.06
15° E	0.11	0.33	2.57	2.16	0.27	8.28
0°	0.19	0.35	2.08	2.76	0.48	6.51
15° W	0.26	0.31	1.40	3.00	0.66	3.84
45° W	3.54	2.95	5.78	34.02	10.68	10.77
(c) Lunar Declination = 18° S						
45° E	0.09	0.16	2.64	0.84	0.21	8.19
15° E	0.09	0.23	2.49	1.32	0.18	7.83
0°	0.11	0.29	2.32	1.92	0.24	7.41
15° W	0.25	0.30	1.43	2.91	0.63	3.84
45° W	0.35	2.91	5.58	33.57	10.47	11.73

^a x is the radial component, y is down range, and z is out of plane.

TABLE II.- 3σ UNCERTAINTIES IN COMPONENTS OF POSITION AND VELOCITY
AT CSI BASED ON CNB, CRO, AND GUA TRACKING DATA

Landing site	3σ position uncertainty, ^a n. mi.			3σ velocity uncertainty, fps		
	$3\sigma_x$	$3\sigma_y$	$3\sigma_z$	$3\sigma_{x^*}$	$3\sigma_{y^*}$	$3\sigma_{z^*}$
(a) Lunar Declination = 18° N						
45° E	0.08	0.15	3.19	0.78	0.21	11.07
15° E	0.12	0.33	3.04	2.19	0.24	10.68
0°	0.20	0.37	2.49	2.91	0.48	8.52
15° W	0.26	0.31	1.54	3.09	0.69	4.59
45° W	3.46	3.00	6.29	33.63	10.59	12.12
(b) Lunar Declination = 0°						
45° E	0.08	0.15	2.85	0.78	0.21	10.02
15° E	0.11	0.31	2.63	2.04	0.24	9.54
0°	0.19	0.35	2.17	2.73	0.48	7.74
15° W	0.26	0.30	1.40	3.00	0.69	4.38
45° W	3.51	3.02	6.10	33.99	10.74	10.86
(c) Lunar Declination = 18° S						
45° E	0.08	0.14	2.63	0.75	0.21	9.30
15° E	0.11	0.29	2.43	1.98	0.24	8.85
0°	0.19	0.33	2.03	2.64	0.45	7.29
15° W	0.26	0.30	1.33	3.00	0.66	4.26
45° W	3.71	3.18	6.10	35.97	11.34	10.38

^a x is the radial component, y is down range, and z is out of plane.

TABLE III.- 3σ UNCERTAINTIES IN COMPONENTS OF POSITION AND VELOCITY
AT CSI BASED ON GST, HAW, AND BDA TRACKING DATA

Landing site	3σ position uncertainty, ^a n. mi.			3σ velocity uncertainty, fps		
	$3\sigma_x$	$3\sigma_y$	$3\sigma_z$	$3\sigma_x^*$	$3\sigma_y^*$	$3\sigma_z^*$
(a) Lunar Declination = 18° N						
45° E	0.11	0.24	4.43	1.11	0.21	12.24
15° E	0.12	0.53	5.06	3.27	0.27	11.64
0°	0.22	0.60	4.38	6.26	0.57	8.16
15° W	0.27	0.45	2.73	3.87	0.72	3.87
45° W	4.91	4.60	5.26	50.37	14.76	39.87
(b) Lunar Declination = 0°						
45° E	0.11	0.18	3.84	0.91	0.21	9.52
15° E	0.12	0.36	4.13	2.28	0.27	9.43
0°	0.18	0.51	3.68	4.30	0.57	6.78
15° W	0.24	0.41	2.75	3.46	0.74	4.61
45° W	4.21	3.75	6.37	42.06	12.48	33.89
(c) Lunar Declination = 18° S						
45° E	0.12	0.15	3.38	0.72	0.21	6.81
15° E	0.13	0.28	3.20	1.68	0.27	6.06
0°	0.16	0.35	3.01	2.37	0.57	5.19
15° W	0.23	0.38	2.76	3.06	0.75	4.95
45° W	3.47	2.90	6.88	33.72	10.23	26.82

^a x is the radial component, y is down range, and z is out of plane.

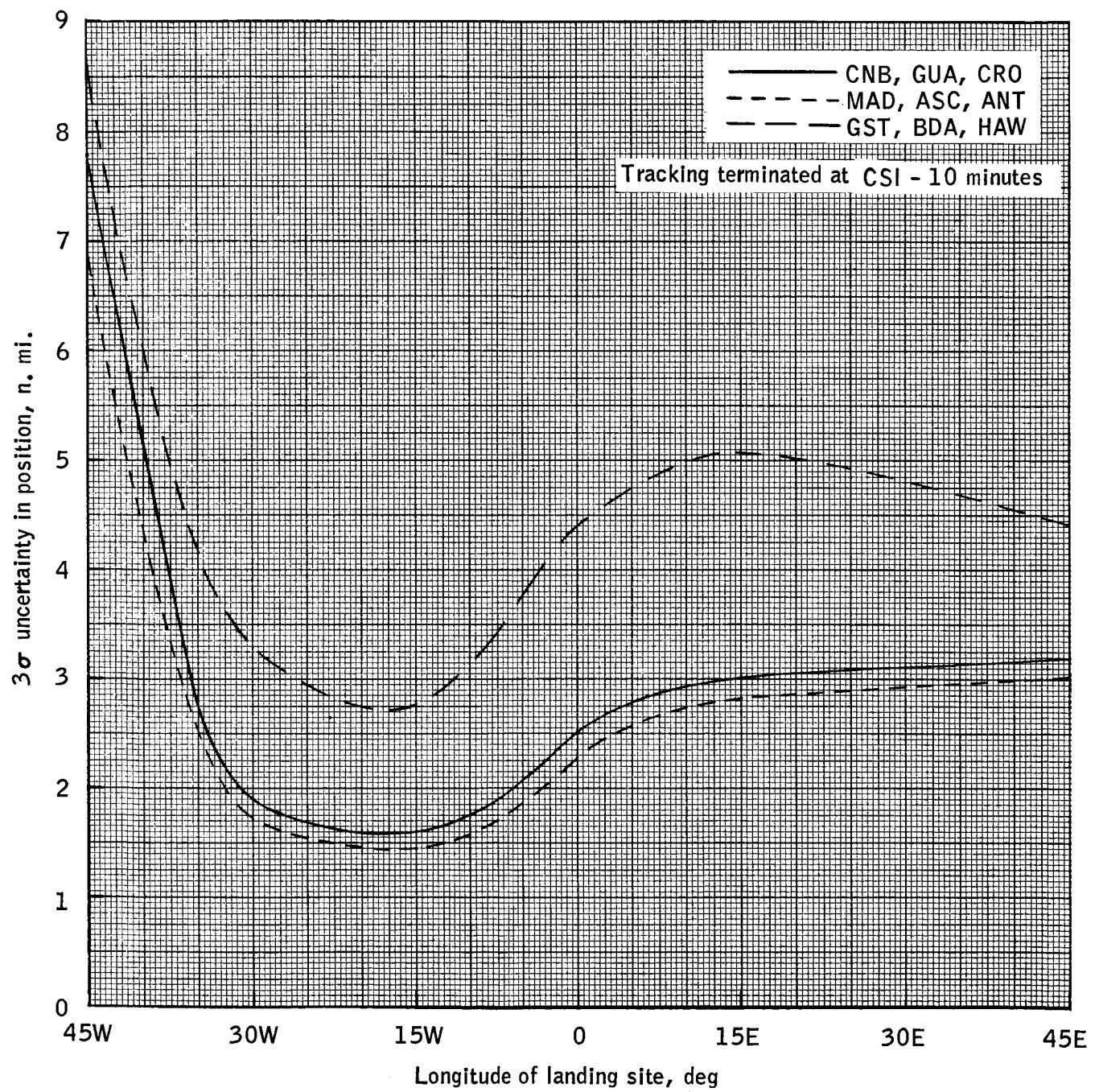


Figure 1. - RSS position uncertainty at CSI for lunar declination of 18° North.

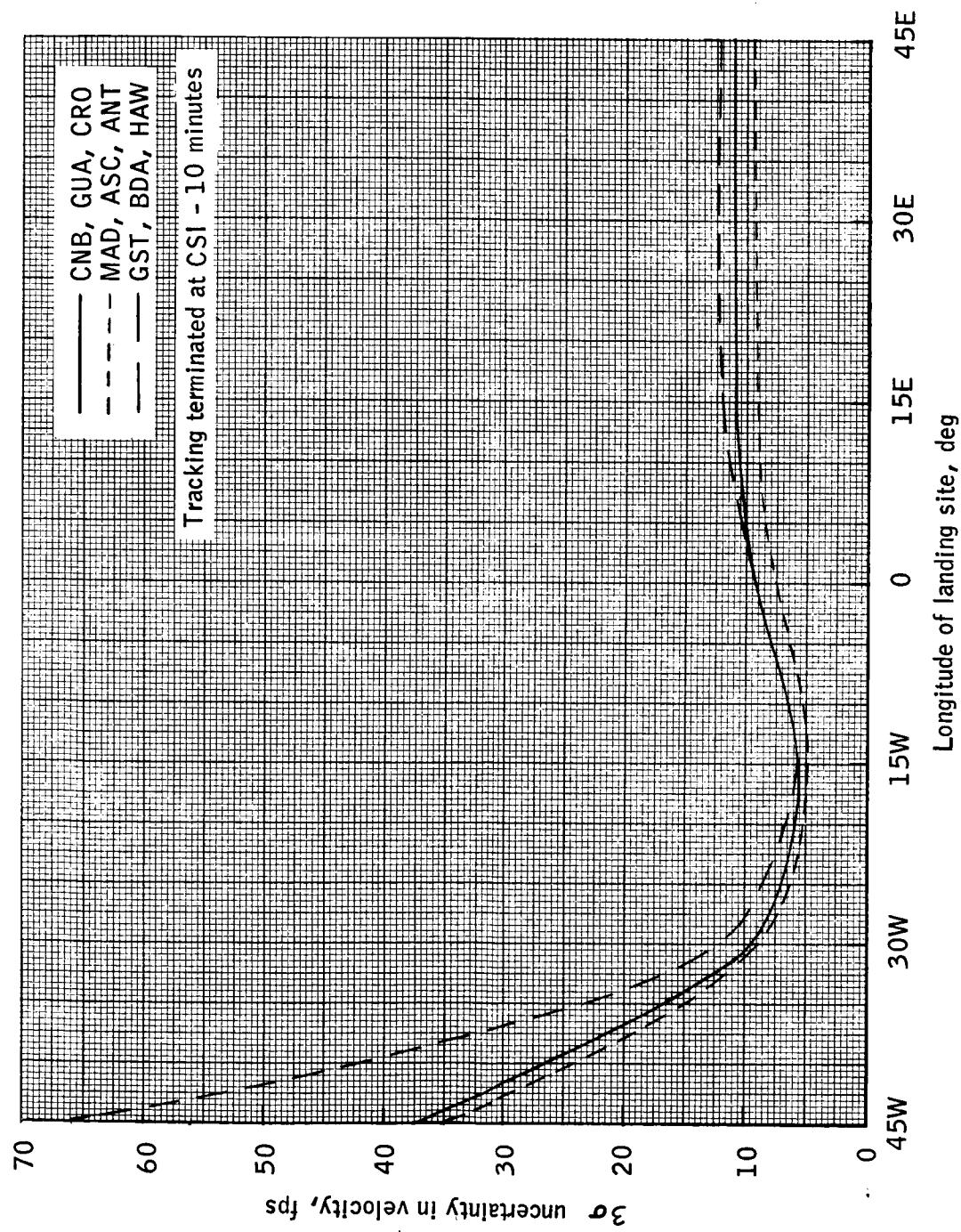


Figure 2.- RSS velocity uncertainty at CSI for lunar declination of 18° North.

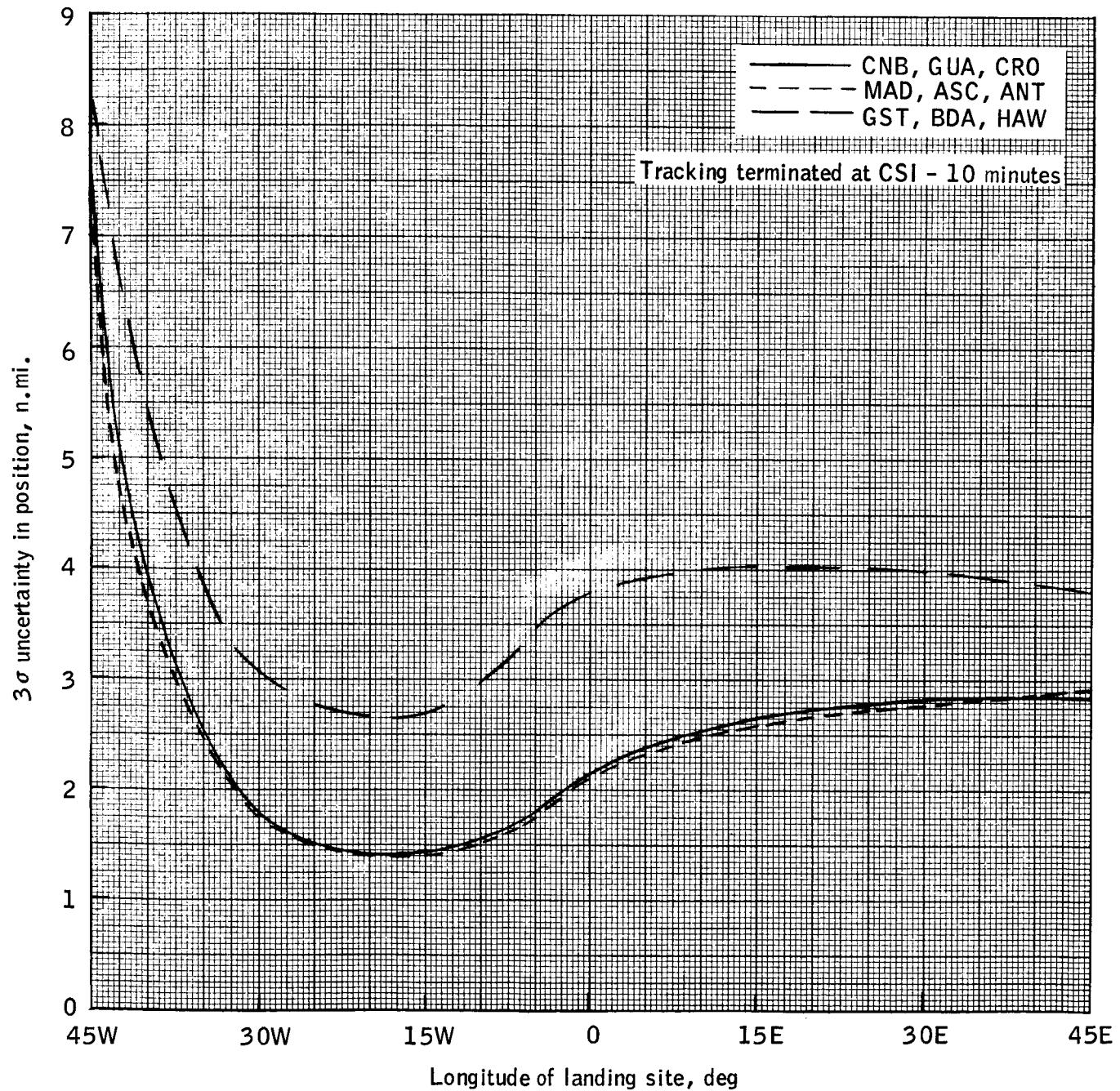


Figure 3. - RSS position uncertainty at CSI for lunar declination of 0°.

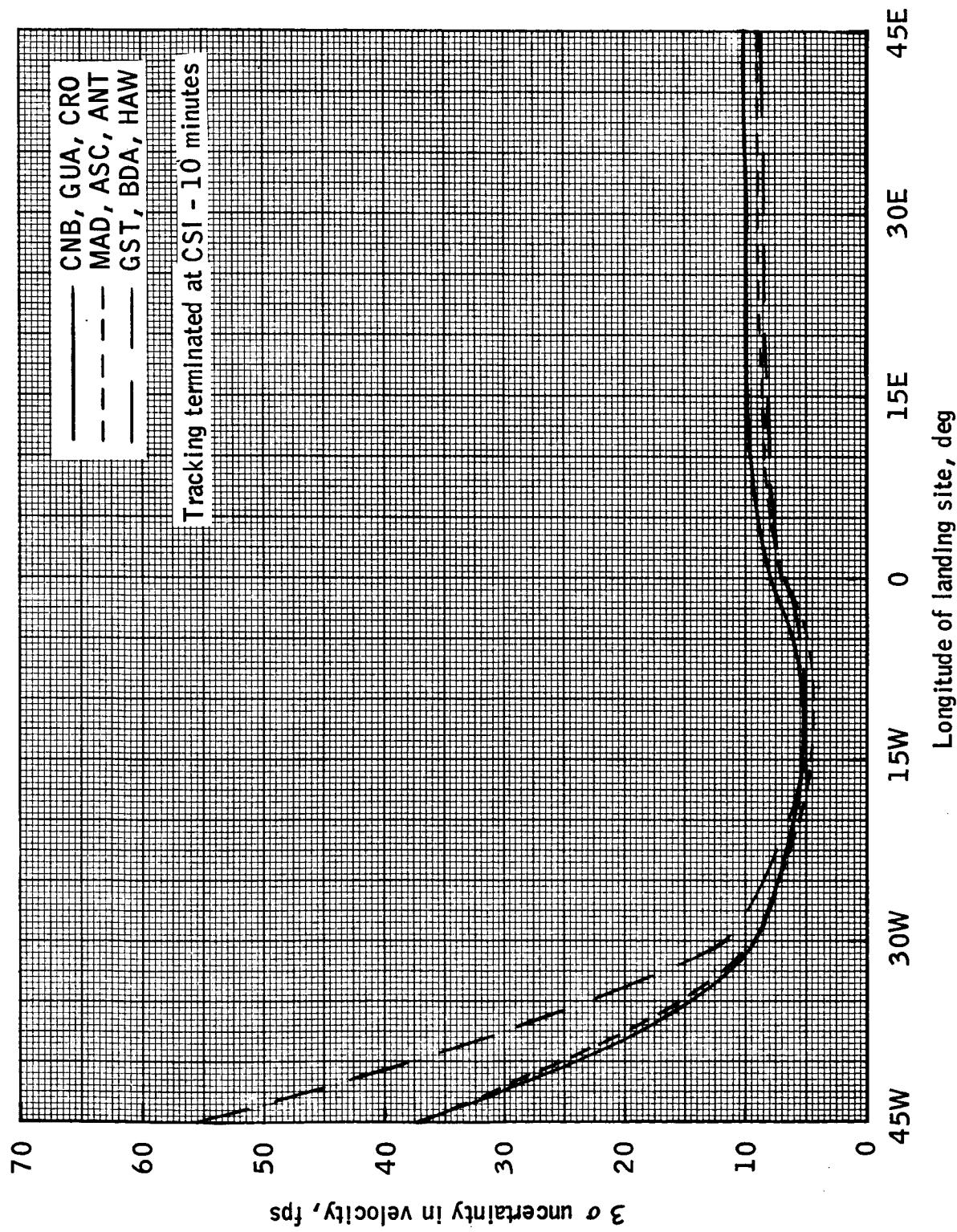


Figure 4.- RSS velocity uncertainty at CSI for lunar declination of 0° .

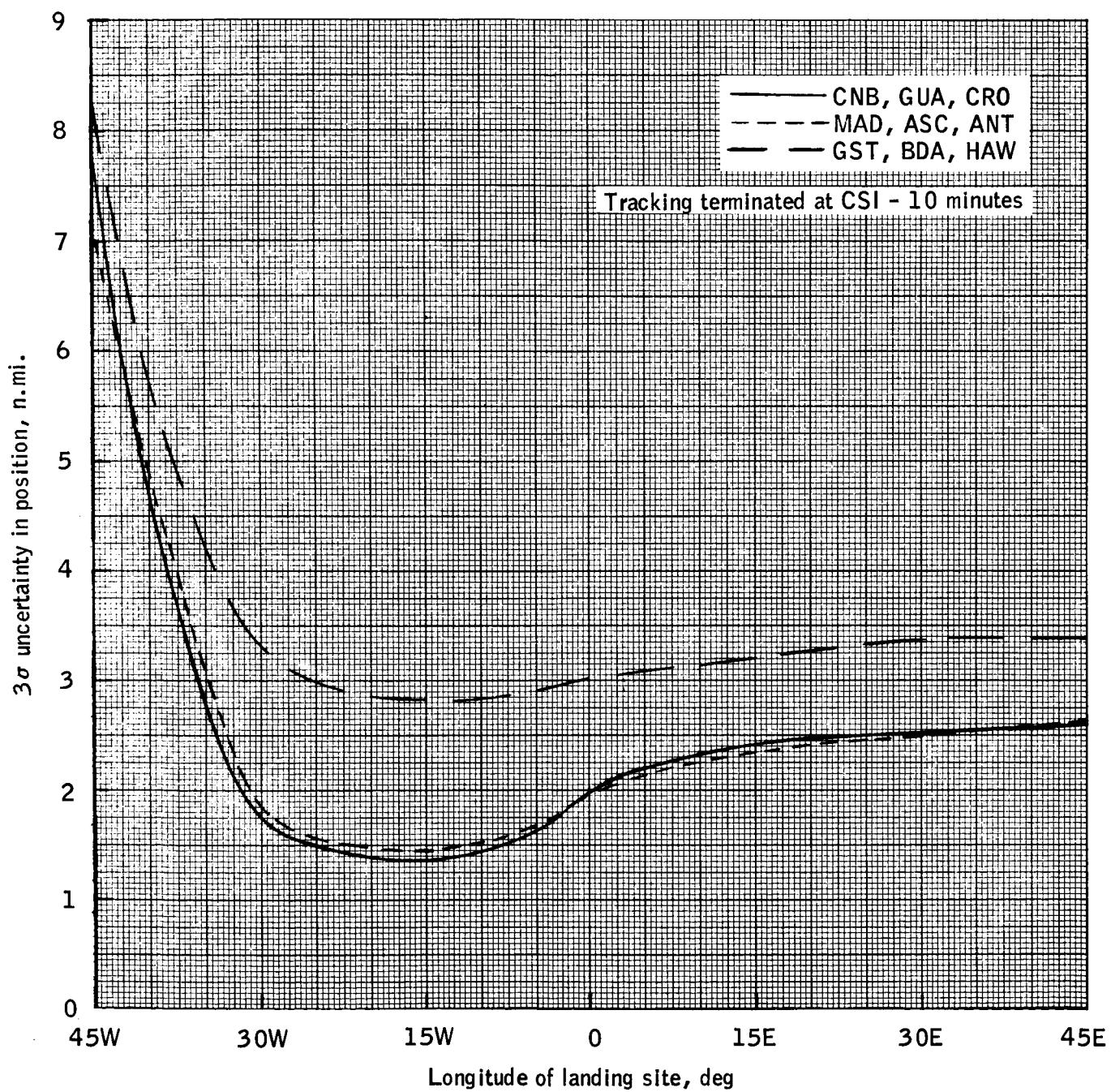


Figure 5.- RSS position uncertainty at CSI for lunar declination of 18° South.

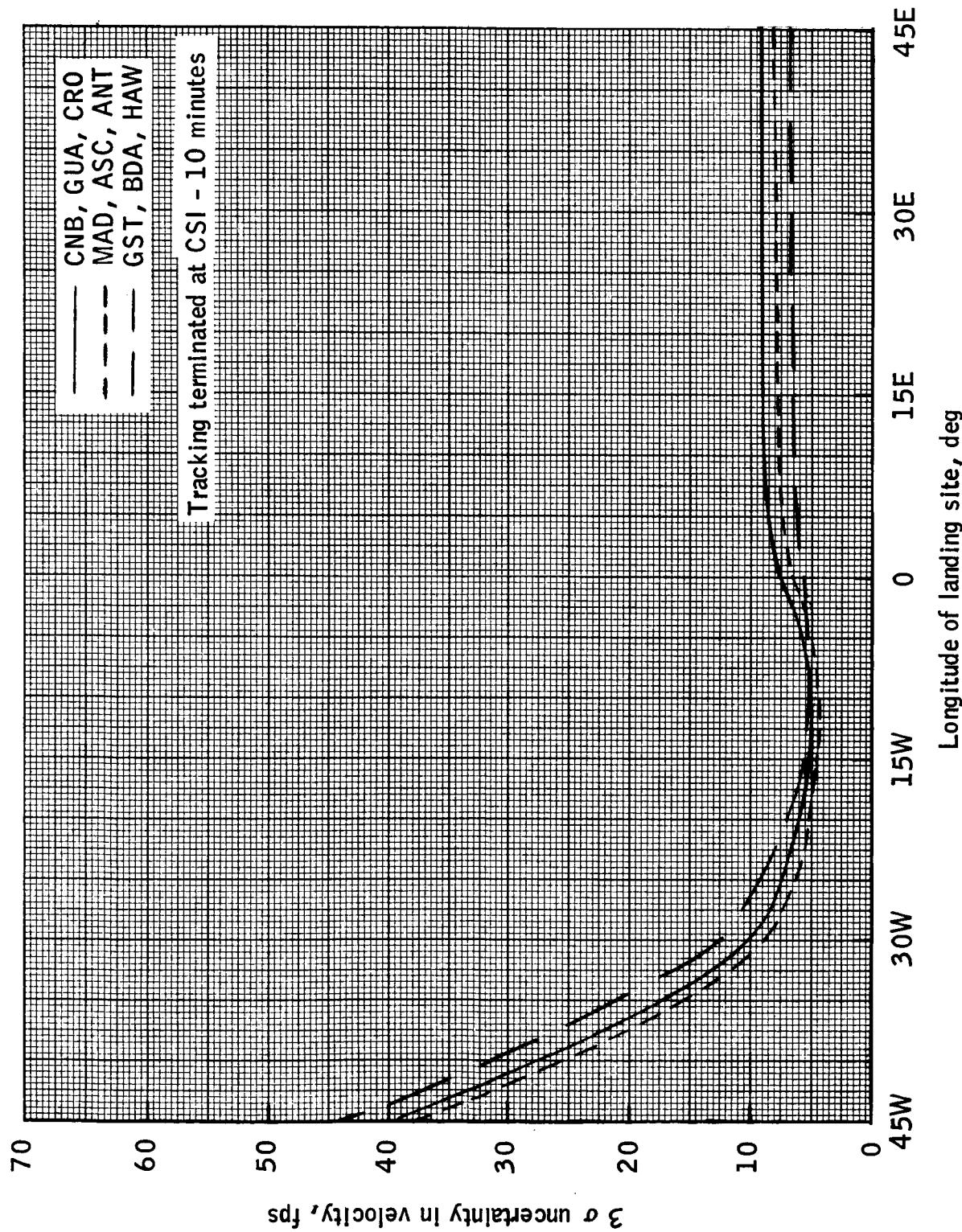


Figure 6. - RSS velocity uncertainty at CSI for lunar declination of 18° South.

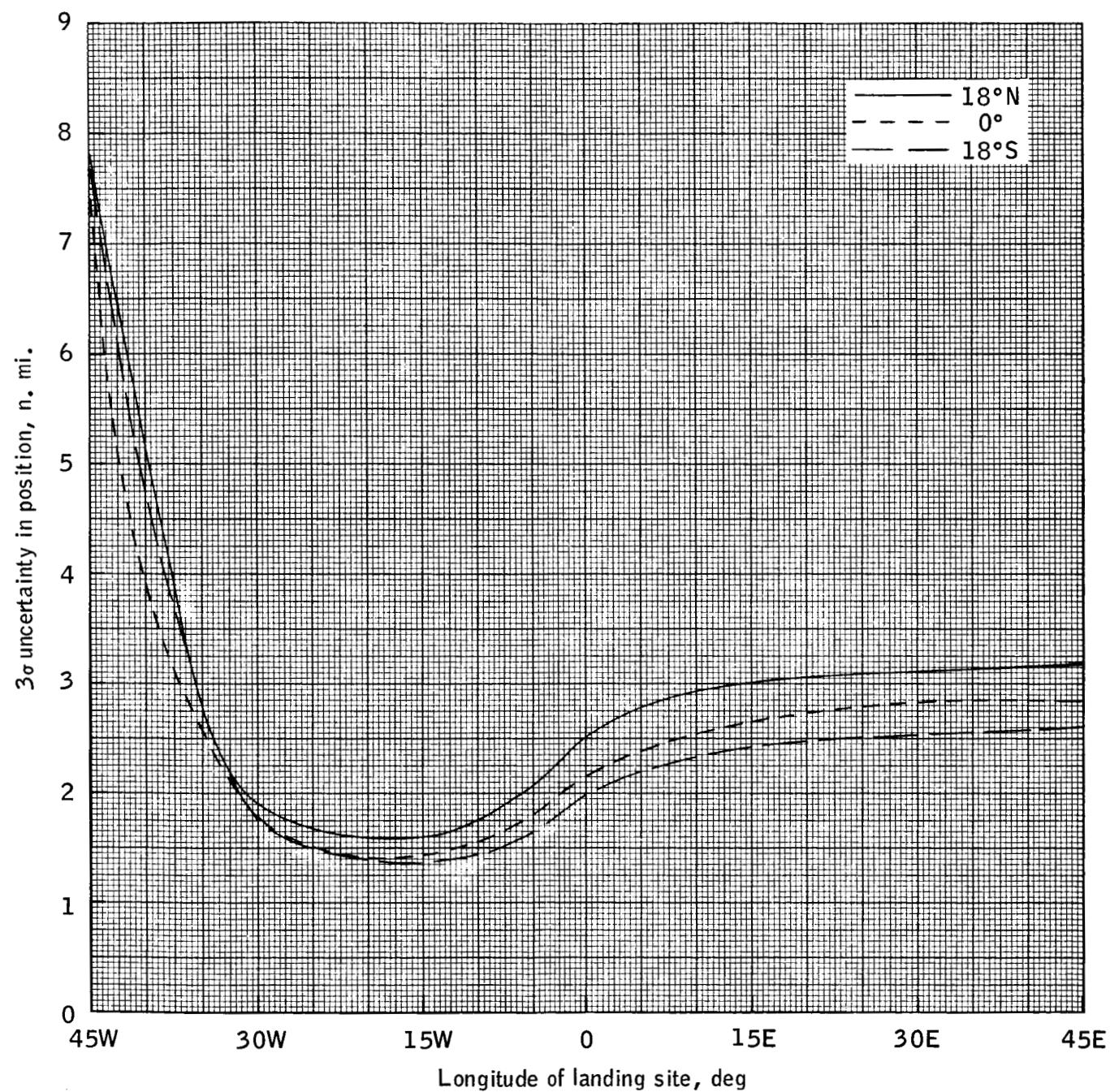


Figure 7.- RSS position uncertainty at CSI for tracking stations Canberra, Guam and Carnarvon.

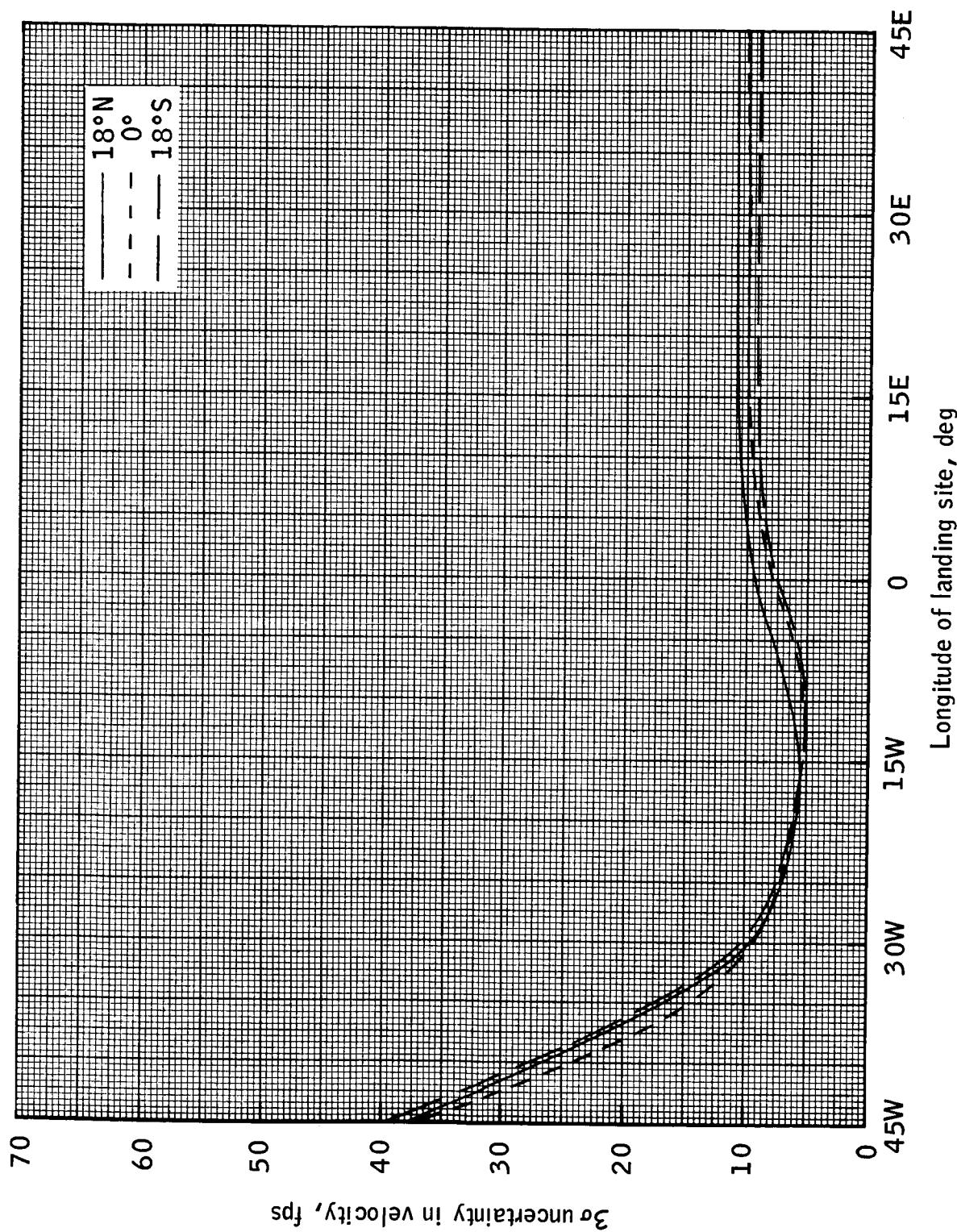


Figure 8.- RSS velocity uncertainty at CS for tracking stations Canberra, Guam and Carnarvon.

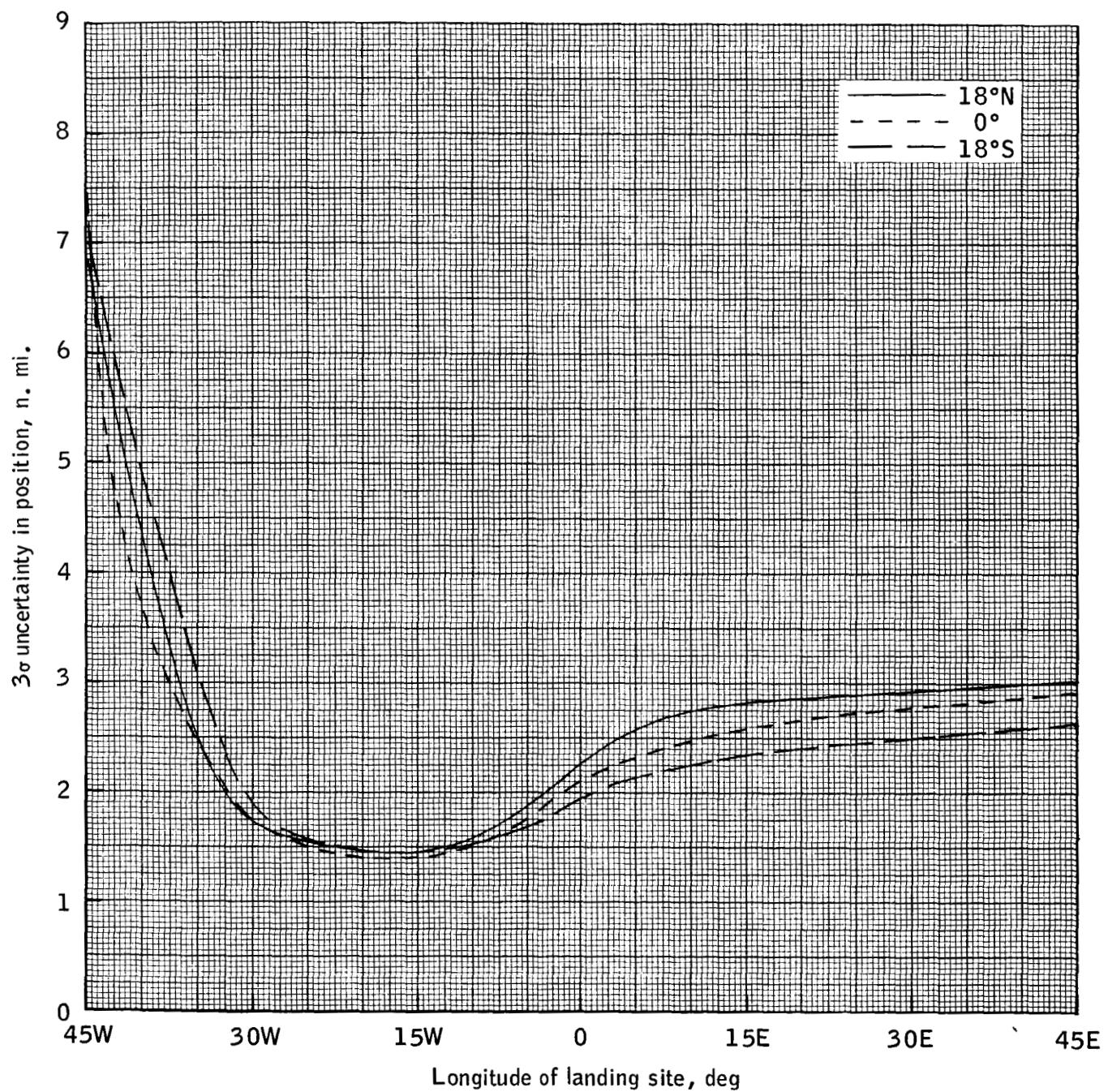


Figure 9.- RSS position uncertainty at CSI for tracking stations Madrid, Ascension and Antigua.

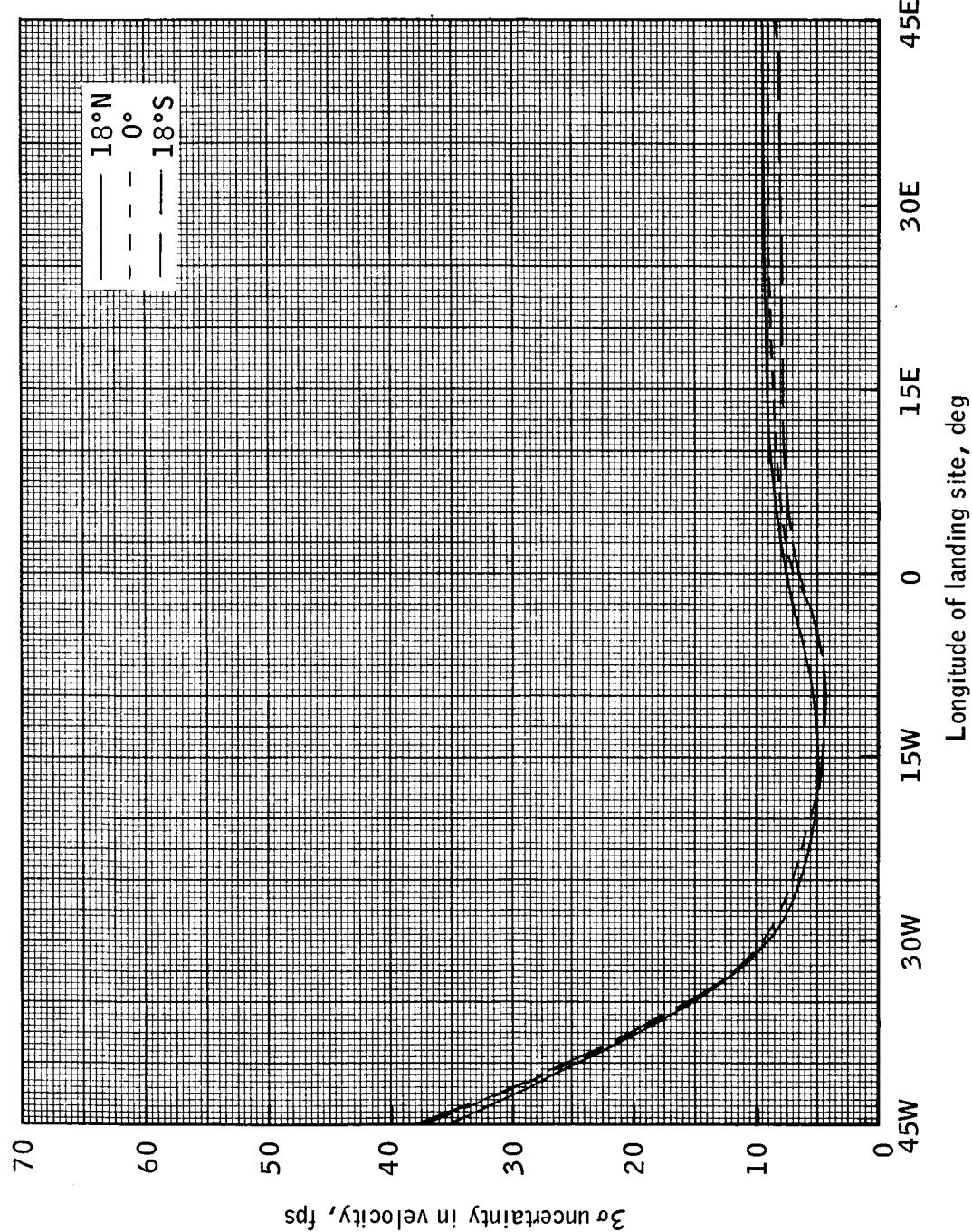


Figure 10. - RSS velocity uncertainty at CSI for tracking stations
Madrid, Ascension and Antigua.

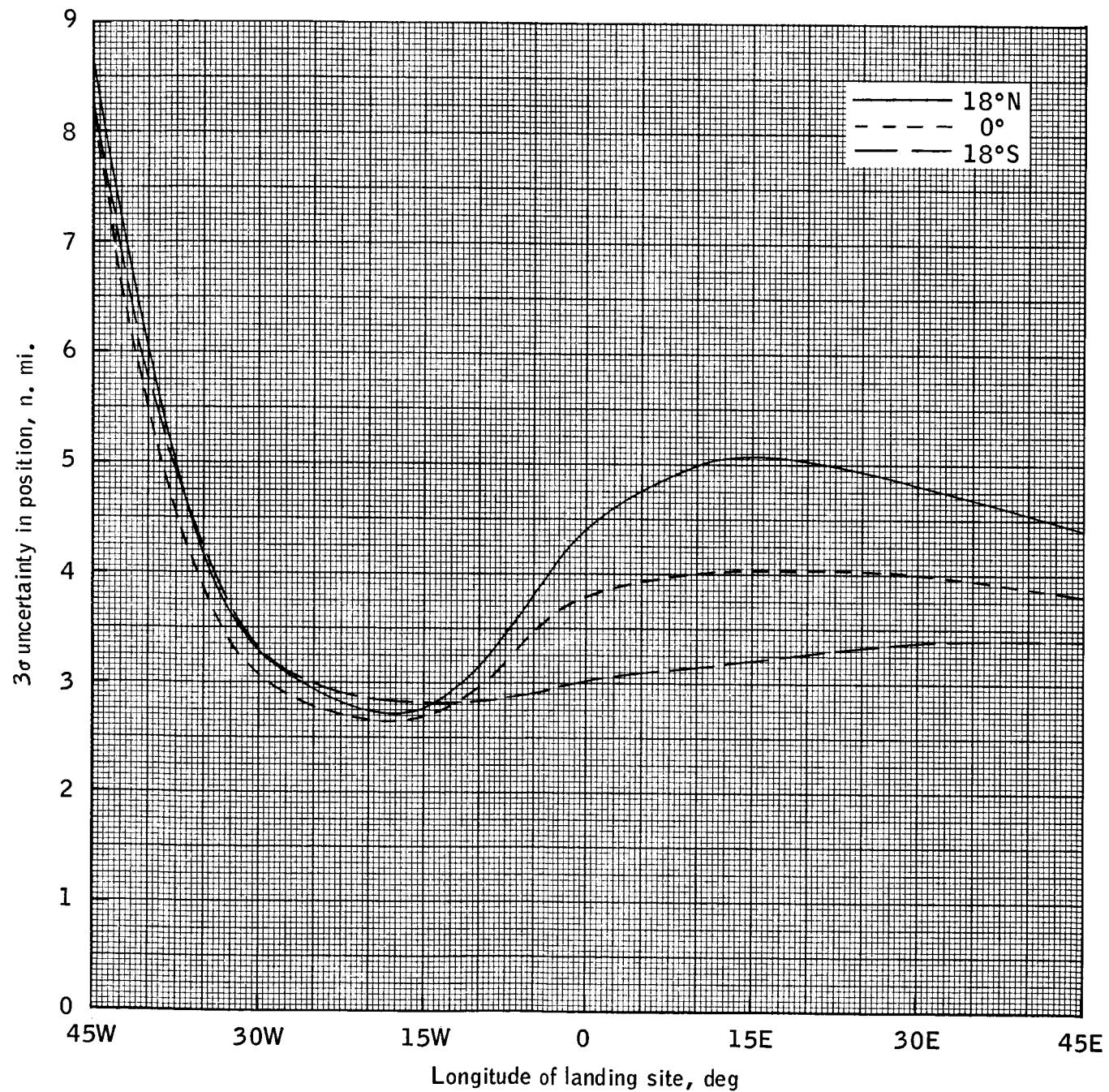


Figure 11.- RSS position uncertainty at CSI for tracking stations Goldstone, Bermuda and Hawaii.

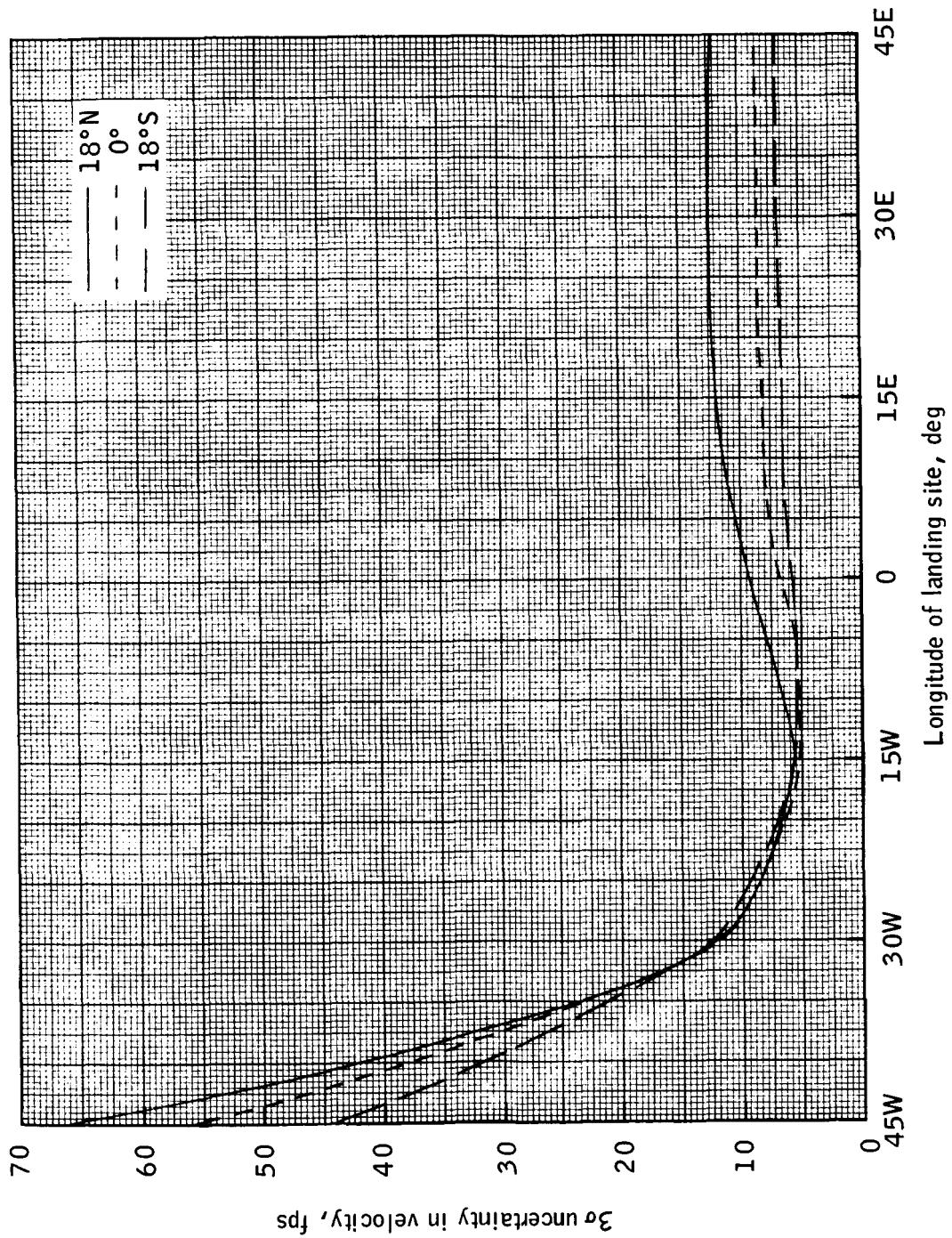


Figure 12.- RSS velocity uncertainty at CSI for tracking stations
Goldstone, Bermuda and Hawaii.

REFERENCE

Mission Analysis Branch: AS-504A Preliminary Spacecraft Reference
Trajectory. MSC Internal Note No. 66-FM-70, Vol. I and II (U).
July 1, 1966. Confidential